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DEVELOPMENT OF A GRAPHITE RESISTANCE HEATER FOR A HYPERSONIC WIND TUNNEL USING NITROGEN

Part III: Analysis of Heater Performance

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Report 527 October, 1960

AF49(638)709

ACKNOWLEDGEMENT

The present study is part of a program of analytical and experimental research in hypersonic gas dynamics being conducted by the Gas Dynamics Laboratory, the James Forrestal Research Center, Princeton University. This research is sponsored by the Office of Aerospace Research, Air Research and Development Command, Fluid Mechanics Division, under Contract AF 49(638)-709, with Lt. H. H. Album as contract officer.

The senior author would like to express his gratitude to Prof. S. M. Bogdonoff, the Office of Aerospace Research and to the British Ministry of Aviation for the opportunity to spend a year at the Gas Dynamics Laboratory, Princeton University.

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SUMMARY

A brief theoretical treatment is given of the problem of calculating the rise in total temperature of a gas which flows through an electrically-heated tube. The theoretical relations obtained are then used to guide the analysis of the results of the experimental development of a graphite resistance heating element in a hypersonic wind tunnel which uses nitrogen as the test gas (see Parts I and II). The analysis indicates that certain simple empirical relations may be used to provide a basis for the assessment of heater performance.

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I. INTRODUCTION

Part I of this report (Reference I) describes a pilot hypersonic wind tunnel which uses nitrogen as the working gas, and Part II (Reference I) describes qualitatively the development of a graphite resistance element for heating the nitrogen. In this Part III, a more detailed description is given of the analysis of the performance of the heater and a selection of results from the experimental program is presented.

From the point of view of the final element as a component of a wind tunnel, it is necessary only to have a graph of the variation of gas total temperature with current for selected values of gas total pressure in order to summarize the heater performance. However, in the present system the gas total temperature is not measured directly but is obtained using a method based on the measurement of mass flows, and this has led to a great deal of scatter and non-repeatability in such simple performance plots (Reference I). Further, in view of the extreme conditions of temperature and pressure which exist together with high electric currents within a very compact element geometry, comparatively few physical measurements can be made with ease in the immediate vicinity of the graphite element.

The performance must, therefore, be described on the basis of the limited information contained in measurements of the inputs and outputs of the system. For these reasons, a simplified theoretical investigation is undertaken to provide at least a guide to the interpretation of the experimental test results.

In Section 2, an idealized heating system is considered and certain theoretical aspects of it are treated briefly. Then in Section 3, the experimental results obtained with the actual heating system used in the pilot nitrogen tunnel are analyzed on the basis of the theoretical formulae . The results of the analysis are used in Section 4 to produce comparable performance curves for two hypothetical idealized heating elements. Some concluding remarks are made in Section 5.

2. THEORETICAL CONSIDERATIONS

2.1 Idealised heating system

The primary purpose of these theoretical considerations is to provide some guidance to the analysis of experimental results. We consider the following idealised heating system. A heating element is constructed from a pure impermeable material in the form of a straight cylindrical tube of circular cross-section. At room temperature there is an adiabatic forced-convection flow of a pure inert thermally-perfect gas through the element; this is called the "cold flow". The mass flow of gas is controlled by a sonic orifice external to the element. An alternating electric current of fixed amplitude is passed through the element which constitutes a wholly resistive load. The total temperature of the gas at the entrance to the element is maintained at room temperature, and the entrance to and the exit of the element and the orifice are maintained at room temperature also. The gas total temperature and the gas total pressure do not vary between the exit of the element and the orifice, and the gas total pressure at the orifice is controlled to be constant at its value in the cold flow. The element is surrounded by a perfectly efficient radiation shield.

It is required to find the relation between the total temperature of the gas at the exit of the element (and hence at the orifice) and the applied current, in terms of the relevant properties of the element material, the element geometry, the gas and the cold flow. A more comprehensive requirement would be for the detailed solution of the steady flow of gas through the element under the heating conditions imposed by the application of the constant current. However, for the present purpose it is necessary to possess only sufficient details of the internal flow to enable a formula for the gas total temperature to be deduced.

We denote the gas total temperature by T_t and the current by i. In non-dimensional form, the independent variable is taken to be $\varkappa$ defined by

$$\varkappa = \frac{r_o i^2}{m_o c_{p_o} T_o} \tag{1}$$

where r_o, m_o and c_{p_o} are the values of the resistance of the element, the mass flow of gas and the specific heat of the gas at room temperature, T_o; the principal dependent variable is taken to be τ defined by

$$\tau = \frac{T_t - T_o}{T_o} \tag{2}$$

2.2 External energy balance

The external energy balance states that the electrical power supplied to the element is equal to the sum of the rate of heat addition to the gas and the rate of external heat loss; there is no external work done.

The electrical power supplied to the element is ri^2 where r is the resistance of the element when the current i is passing; we introduce the non-dimensional quantity λ defined by

$$\lambda = \frac{r i^2}{m_o c_{p_o} T_o} \tag{3}$$

The rate of heat addition to the gas is the increase of total enthalpy per unit time, namely $m(h_t - h_o)$ where m is the mass flow when the current i is passing and $(h_t - h_o)$ is the increase of the specific total enthalpy of the gas from the entrance to the exit of the element; we introduce the non-dimensional quantity μ defined by

$$\mu = \frac{m(h_t - h_o)}{m_o c_{p_o} T_o} \tag{4}$$

Now the mass flow is given (see Part I) by the formula

$$m = \frac{\Gamma A^* p_t}{(RT_t)^{\frac{1}{2}}} , \tag{5}$$

where A^* is the orifice area, p_t is the total pressure at the orifice, R is the gas constant for unit mass and Γ is a factor which is taken to be effectively constant numerically. The cold mass flow is, therefore,

$$m_o = \frac{\Gamma \; A^*_o \; p_{t_o}}{(RT_o)^{\frac{1}{2}}} \tag{6}$$

The total pressure p_t is controlled to be equal to p_{t_o} for any current, and it is assumed that the orifice area does not change with gas temperature, that is we take

$$A^* = A_o^* \tag{7}$$

Hence it follows that

$$\frac{m}{m_o} = \left(\frac{T_o}{T_t}\right)^{\frac{1}{2}} = (1+\tau)^{-\frac{1}{2}} \tag{8}$$

Also, for a thermally-perfect gas, the quantity $(h_t-h_o)/c_{p_o}T_o$ is a function of τ only, and for the range of temperatures of interest we may assume that it can be represented analytically by the formula

$$\frac{(h_t-h_o)}{c_{p_o}T_o} = \tau \quad (1+k\tau) \quad , \tag{9}$$

where k is a small constant which represents the departure of the gas from a calorically-perfect gas; a study of the enthalpy of nitrogen (from the data of Reference 2) leads to the choice $k = 0.0185$ with $T_o = 530^oR.$, for this gas. Therefore, it follows that for a given gas μ depends on τ only and can be calculated once and for all as a function of τ:

$$\mu = (1+\tau)^{-\frac{1}{2}} \tau (1+k\tau) \quad ; \tag{10}$$

the variation of μ with τ for nitrogen is shown in Figure 1.

The rate of external heat loss is given, in the absence of losses due to radiation, by the rate at which heat is conducted away from the ends of the element; introducing the non-dimensional quantity ν defined as the rate of external heat loss divided by $m_o c_{p_o} T_o$, we have

$$\nu = \frac{\zeta_o A_E}{m_o c_{p_o} T_o} \left(\left|\frac{dT_E}{dx}\right|_{x=o} + \left|\frac{dT_E}{dx}\right|_{x=\ell} \right) , \qquad (11)$$

where ζ_o is the thermal conductivity of the element material at room temperature*, A_E is the cross-sectional area of the element (not including the area of the gas passage), and $\left|dT_E/dx\right|_{x=o}$ and $\left|dT_E/dx\right|_{x=\ell}$ are the temperature gradients along the element at the entrance and the exit; the length of the element is denoted by and x denotes distance along the element from the gas entrance. We assume that the element temperature T_E increases from T_o at the gas entrance to a maximum $T_{E_{max}}$ and then decreases to T_o at the gas exit. By introducing the non-dimensional quantities τ_E, $\tau_{E_{max}}$ and ξ defined by

$$\tau_E = \frac{T_E - T_o}{T_o} , \qquad (12)$$

$$\tau_{E_{max}} = \frac{T_{E_{max}} - T_o}{T_o} , \qquad (13)$$

$$\xi = \frac{x}{\ell} , \qquad (14)$$

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* For graphite, there exists a rough but useful relation (Ref. 3) which connects the room temperature values of thermal conductivity ζ_o and resistivity ρ_o of many different grades of graphite; in the units used in the present investigation the relation is

$(\zeta_o$ watts per inch $^oR.)(\rho_o$ ohm inches$)$ = 0.00130 watt ohms per ^oR.

we write the element temperature distribution in the form

$$\tau_E = \tau_{E_{max}} f(\xi) \qquad , 0 \leq \xi \leq 1 \quad , \tag{15}$$

where $f(\xi)$ is unspecified except that $f(0) = 0 = f(1)$ and $f_{max} = 1$. It then follows that

$$\nu = \frac{\zeta_0 A_E}{m_o c_{p_o} \ell} (f'(0) - f'(1)) \tau_{E_{max}} \quad . \tag{16}$$

We now connect the gas total temperature and the element maximum temperature by writing

$$\tau_{E_{max}} = \frac{\tau}{\chi} \qquad , \tag{17}$$

where χ depends on τ and the various non-dimensional parameters of the system. It then follows that

$$\nu = \frac{\zeta_0 A_E}{m_o c_{p_o} \ell} \left((f'(0) - f'(1)) \, 1/\chi \right) \tau \quad , \tag{18}$$

and this expression is simplified by writing

$$(1/\alpha - 1) = \frac{\zeta_0 A_E}{m_o c_{p_o} \ell} \left((f'(0) - f'(1)) \, 1/\chi \right) \quad , \tag{19}$$

so that we have

$$\nu = (1/\alpha - 1)\tau \tag{20}$$

It may be remarked that the quantity α is related to the efficiency of the heating system η which is defined as the ratio of the heat received by the gas to the energy supplied to the element and is, therefore, given by

$$\eta = \mu/\lambda \tag{21}$$

Since the external energy balance gives the relation

$$\lambda = \mu + \nu \tag{22}$$

It follows from equations (21), (22), (10) and (20) that η and α are connected by

$$\eta = \frac{1 + k\tau}{1 + k\tau + (1/\alpha - 1)(1 + \tau)^{\frac{1}{2}}} \tag{23}$$

If η_0 denotes the limiting value of η in the cold flow, follows that

$$\lim_{\tau \to 0} \alpha = \eta_0 \quad , \tag{24}$$

so that the initial value of α is the initial efficiency of the system.

For subsequent use, the most convenient expression of the external energy balance is

$$\lambda = \mu + (1/\alpha - 1)\tau \tag{25}$$

It is recalled that for a given gas μ depends on τ only; α is regarded as a quantity which depends on τ and on the non-dimensional parameters of the system.

2.3 Relation between gas total temperature and current

The independent variable $\varkappa$ defined by equation (1) is connected with the quantity λ by the relation

$$\lambda = \beta \varkappa \tag{26}$$

where β is given by

$$\beta = \frac{r}{r_o} \quad ; \tag{27}$$

β is regarded as a quantity which depends on τ and the non-dimensional parameters of the system. The form of β may be obtained from the relations (15) and (17) used in expressing the required features of the solution of the internal flow problem. If the resistivity of the element material at room temperature is denoted by ρ_o, and the variation of the resistivity ρ is written in the form

$$\rho/\rho_o = g(\tau_E) , \tag{28}$$

then it follows, since the area of the element is constant, that

$$r_o = \frac{\rho_o \, \ell}{A_E} , \tag{29}$$

$$r = \frac{\rho}{A_E} \int_0^{\ell} g(\tau_E)\,dx \quad . \tag{30}$$

Hence, by using equations (15) and (17), we have

$$\beta = \int_0^1 g\,(\tau/\gamma\; f(\xi)\;)\;d\xi \quad . \tag{31}$$

By combining equations (25) and (26) we obtain the form of the required relationship between τ and λ ;

$$\lambda = 1/\beta\;[\mu + (1/\alpha - 1)\;\tau\;] \tag{32}$$

where, for the idealised system considered, α and β are given by equations (19) and (31), respectively.

3. ANALYSIS OF EXPERIMENTAL RESULTS

3.1 Differences between idealised and actual heating systems

Element material

The idealised material is pure and impermeable, and has known physical properties. The graphites used in the present tests were neither pure nor impermeable, and their physical properties were not known accurately enough quantitatively to be usable in calculating heater performance. The purity of the graphite depended on the grade; some grades contained noticeable amounts of heterogenous ash in addition to the carbon compounds ordinarily present in commercial graphites. The permeability of the graphite also depended on the grade. There is a further complication in the fact that the purity, porosity and physical properties of the graphites varied slightly from piece to piece within the same grade.

Element geometry

By comparison with the simple cylindrical tube considered as the idealised heating element, the actual elements had very complicated geometries. The cross-sectional area of the solid part of the element was not constant along the entire length because of the special shapes used for the end contacts. The area of the gas passage was constant over most of the element length, but variations occurred near the entrance and exit. The passage was not straight-through but in the form of a spiral, and it was rectangular and not circular in cross-section. It is, therefore, difficult to relate such a geometrical shape to a simple tube. Undoubtedly, the formulae deduced for the conduction losses from the idealised element and the resistance of the idealised element are not applicable to the actual element.

Gas

The idealised gas is pure and inert and thermally-perfect. The actual gas was nominally pure but nevertheless it contained small properties of other substances and these impurities rendered it not inert to reaction with graphite. Three brands of nitrogen, of increasing purity, were used during the tests reported here. It must be remarked also that nitrogen itself is only approximately thermally-perfect at the highest pressures involved in the tests. In addition, occasionally the gas in the actual system was contaminated with extraneous substances.

Cold flow

On the whole, the cold flow of gas was controlled as accurately as was desired, except that a slight drop in gas total temperature during a run was possible. There was not likely to have been any erosion of the gas passage. The measured value of mass flow was subject, however, to the accuracy of the calibration of the flow meter, and in the present tests the flow meter was modified once in order to facilitate the measurement of the cold mass flow. Unfortunately, it was not possible to measure all the cold flow parameters involved in the theory, because the cold resistance of the element was sometimes obscured by the contact resistances at the ends.

Hot flow

The end conditions on the gas total temperature and element temperature is assumed in the idealised system were probably not precisely reproduced in the actual system because the cooling system applied to the ends of the element and to the orifice may not have been sufficiently effective to reduce the temperatures there to exactly room temperature. Also, the radiation shield used in the tests was not likely to have been perfectly efficient in eliminating all radiation losses. The element was effectively a purely resistive load.

3.2 Procedure for analysis of results

The principle of the analysis procedure is as follows: From the measurements of the current and the gas total temperature (the latter obtained indirectly from the mass flow) and the cold flow parameters it is possible to calculate λ and τ, and to make a plot of experimental τ against experimental λ. We would like to compare with the experimental points a smooth theoretical curve, such as that given by equation (32) for the idealised heating system. However, the theoretical treatment of the idealised system itself is very incomplete, and when cognizance is taken of the differences between the actual and idealised systems, it is clear that the foregoing theoretical considerations can serve only as a guide to the analysis of the present experimental results. We consider that the effect on equation (32) of the differences between the actual and idealised system is to alter, in a possibly drastic manner, the expressions (19) and (31) for the quantities α and β respectively in terms of τ and the cold flow parameters, and also to limit, because of premature element failure, the range of applicability of this equation. However, we assert that the form of equation (32) still holds for the actual system. We note that it is possible to obtain experimental values of α and β by measuring the voltage v across the element for each current setting: firstly, λ may be calculated from

$$\lambda = \frac{iv}{m_o c_{p_o} T_o} \qquad , \qquad (33)$$

and then α is given by the formula, obtained from equation (25),

$$\alpha = \frac{1}{1 + \left| \frac{\lambda - \mu}{\tau} \right|} \qquad ; \qquad (34)$$

secondly, r is calculated from

$$r = \frac{v}{i} \qquad , \qquad (35)$$

and then β may be obtained from equation (27), namely $\beta = r/r_o$. The aim of the analysis then becomes to examine the experimental values of the values of the quantities α and β for evidence of consistency and particular significance which may lead to the formulation of an empirical relation connecting $\varkappa$ and τ.

In fact, in analysing the present results, the procedure cannot be carried out precisely as described because it was not possible to measure r_o accurately enough in the cold flow, and therefore, it is not immediately possible to calculate $\varkappa$ and β. In order to deduce r_o, an auxiliary analysis is made of the values of the element resistance r. We assume that the variation of β with τ can be represented by an equation of the form

$$\beta = (1 + \tau)^{-\frac{1}{2}} (1 + a\tau + b\tau^{3/2}) \qquad , \qquad (36)$$

so that the resistance is given by

$$r = r_o \, (1 + \tau)^{-\frac{1}{2}} \, (1 + a\tau + b\tau^{3/2}) \quad , \tag{37}$$

From a plot of r against τ, it is possible to estimate the minimum resistance r_{min} obtained during a run, and we define the quantity ψ by

$$\psi = (1+ \tau)^{\frac{1}{2}} \, r/r_{min} \tag{38}$$

It follows that our assumed form for ψ is

$$\psi = r_o/r_{min} \, (1 + a\tau + b\tau^{3/2}) \quad . \tag{39}$$

The quantity ψ can be calculated for each current setting and the values plotted against τ. A smooth curve is drawn between the experimental points, so as to cover the range $1 \leq \tau \leq 4$; let ψ_1, ψ_2, ψ_3 be the values of ψ on the curve at $\tau_1 = 1$, $\tau_2 = 9/4$, $\tau_3 = 4$; then it follows that r_o/r_{min} is given by

$$r_o/r_{min} = 1/13 \, (36 \, \psi_1 - 32 \, \psi_2 + 9 \, \psi_3) \quad , \tag{40}$$

and hence r_o can be found. It may be noted also that a and b are given in terms of ψ_1, ψ_2 and ψ_3 by

$$a \, r_o/r_{min} = - 1/13 \, (37 \, \psi_1 - 56 \, \psi_2 + 19 \, \psi_3) \quad , \tag{41}$$

$$b \, r_o/r_{min} = 1/13 \, (14 \, \psi_1 - 24 \, \psi_2 + 10 \, \psi_3) \quad , \tag{42}$$

so that this method of extrapolating the resistance measurements to find r_o also yields the coefficients in the assumed form (36) for β.

3.3 Some selected results

Lists of the elements tested in the three phases of the present investigation are given in Sections A.1, A.2 and A.3 of the Appendix. The lists include information on the values taken by the basic cold flow parameters; the parameter ω'_o, not previously defined, is a measure of the drop of total pressure down an element in the cold flow, and is regarded as an indication of the internal geometry of the element. The analysis of the results of only a few tests is presented here. The reasons for the exclusion of the results of the other tests are given in Section A.4 of the Appendix. It may be remarked that the omitted results are not in any way inconsistent with those presented, which are selected for convenience and clarity.

Results for α

Figure 2 shows the variation of α with τ for Heater 1, which was tested without a radiation shield. It is noticeable that α decreases with increase of τ, thus indicating an increase in heat loss with increase of temperature which is consistent with the existence of radiation losses at higher temperatures.

Figure 3 shows the variation of α with τ for Heaters 9, 10 and 11, all of which were of nearly identical geometry (Regular) and material (NT0005) and which were tested in the presence of a radiation shield. These results show repeatability between different runs of the same heater, near-constancy of α for each heater, and repeatability between different heaters. The repeatability of the results is gratifying but is to be expected in comparable tests. The near-constancy of α is of a more surprising nature, since there is no obvious reason why α should be independent of τ. The trend for small τ is obscured by the scatter, which is of the magnitude to be expected from the errors involved in calculating α, which may be shown to become infinite as $\tau \longrightarrow 0$ when there is an error in mass flow measurement. The tendency of α to increase for $\tau > 5$ is an indication of the failure of the element leading to partial blocking of the orifice and a consequent erroneous measurement of mass flow.

Figure 4 shows the variation of α with τ for Heaters 12, 13, 14 and 15, which were of nearly identical geometry (Regular) but of various materials (12: NT0005; 13, 14: A.T.J.; 15: RT0003). These results show moderately good repeatability between Heater 12 and the mean value for Heaters 9, 10 and 11, and between Heaters 13 and 14, but there is an unexplained discrepancy between the results of the two runs of Heaters 15. It is noticeable again, however, that α is nearly independent of τ in each run. Failure of the elements occurred at lower temperatures than for Heaters 9, 10 and 11.

Figure 5 shows the variation of α with τ for Heaters 16 and 17, which were of nearly identical geometry (Short) but of different materials (16: RT0004; 17: RT0003). Repeatability between the two runs of Heater 16 is good, and the results for Heater 17 are quite close to those for Heater 16. Again, α is roughly independent of τ. These short elements appear to fail at higher temperatures than the corresponding regular elements. It is somewhat surprising, since α is a measure of the efficiency of the system, that the mean value of α for the short elements is so near to the mean value of α for the corresponding regular elements.

Results for β

Figure 6 shows the variation of β with τ for Heaters 9, 10 and 11. The values of r_o listed in the Appendix were obtained by performing the auxiliary analysis described in the previous section. This analysis gave r_o/r_{min} = 1.72 for all the heaters. Also, it followed that a = 0, b = 0.0385, and the curve for β from equation (36) is also shown in this figure. Repeatability between tests is seen to be quite satisfactory.

Variation of τ with $\varkappa$

Figure 7 shows the variation of τ with $\varkappa$ for Heaters 9, 10 and 11. The values of $\varkappa$ were calculated using the values of r_o deduced from the auxiliary analysis. The analytical curve shown was obtained by taking the mean value α = 0.8 (from Figure 3) and the analytical curve for β (see Figure 6) and substituting in equation (32). The scatter at the higher values of $\varkappa$ indicates that the elements were failing when the measurements were being taken.

4. EMPIRICAL CORRELATION OF HEATER PERFORMANCE

The foregoing analysis indicates that, despite the various approximations and assumptions involved in the derivation of heater performance, the experimental variations of α and β with τ are consistent and fairly simple. By smoothing the results for α and β an empirical performance curve for τ against $\mathcal{H}$ can be calculated. We now make use of the trends suggested by the experimental results to calculate hypothetical performance curves for idealised elements constructed of two fairly extreme grades of graphite for which the resistivities are known. The hypothetical cases are so constructed that they give a rough comparison with the empirical performance curve given in Figure 7 for Heaters 9, 10 and 11 (NT0005 graphite, Regular geometry).

On the basis of the foregoing results, we take α to be independent of τ; we do not assert that this is likely to be a genuine result but merely that such an assumption is likely to give an adequate representation over a fairly wide range of τ. For comparison with the tests of Heaters 9, 10, and 11 we select $\alpha = 0.8$.

The two grades of graphite considered are graphitised lampblack base and graphitised petroleum coke base. The values of ρ_0 and the variations of $\rho/\rho_0 = g(\tau_E)$ with local temperature for these graphites (see Reference 4) are shown in Figure 8; the form of $g(\tau_E)$ for lampblack base graphite can be fitted with very good accuracy by the formula

$$g(\tau_E) = (1 + \tau_E)^{-\frac{1}{2}} \quad ; \qquad (43)$$

a sufficiently simple analytical expression for $g(\tau_E)$ for petroleum coke base graphite is not available. In order to calculate β from equation (31), it is assumed that the function $f(\xi)$ involved in equation (15) for the element temperature distribution possesses a type of symmetry expressed by

$$f(\xi) = \begin{cases} \varphi(\xi/\sigma) , & 0 \leq \xi \leq \sigma , \\ \varphi\left|\dfrac{1-\xi}{1-\sigma}\right| , & \sigma \leq \xi \leq 1 , \end{cases} \qquad (44)$$

where σ denotes the value of ξ at which $f(\sigma) = f_{max} = 1$; the function $\varphi(\theta)$ satisfies $\varphi(0) = 0$, $\varphi(1) = 1$, $\varphi'(1) = 0$. Then equation (31) for β reduces to

$$\beta = \int_0^1 g\left(\frac{\tau}{\chi} \varphi(\theta) \right) d\theta , \qquad (45)$$

which is independent of σ. We now take, for simplicity, for $\varphi(\theta)$ the specific parabolic form

$$\varphi(\theta) = \theta(2 - \theta) \qquad (46)$$

In this case, equation (45) can be integrated analytically for the form of $g(\tau_E)$ for lampblack base graphite as given in equation (43), giving the result

$$\beta = (\tau/\chi)^{-\frac{1}{2}} \sin^{-1} \left[(\tau/\chi)^{\frac{1}{2}} (1 + \tau/\chi)^{-\frac{1}{2}} \right] ; \qquad (47)$$

for petroleum coke base graphite the integration is performed numerically.

Next, on the basis of the results of Phase 3 of the present investigation, as recorded in Part II of this report (Reference 1), we assume that χ is independent of τ. While we have no actual measurements of χ for Heaters 9, 10, and 11, it appears from the results of Phase 3 that it is reasonable to choose $\chi = 0.8$.

The forms of β for the two hypothetical idealised elements, in the case when $\chi = 0.8$, are shown in Figure 9, where the experimentally-based curve for Heaters 9, 10 and 11 is also shown. It appears that NT0005 is a material more nearly akin to lampblack base graphite than to petroleum coke base graphite, especially at low temperatures. Incidentally, this indication is in agreement with the extrapolated values of r_o deduced for these heaters, for which a value of ρ_o of the magnitude of that for lampblack base graphite is required for consistency with the size of the regular element.

To obtain the corresponding performance curves for the hypothetical elements, it is necessary to substitute $\alpha = 0.8$ and β as given in Figure 9 into equation (32). The resultant performance curves are shown in Figure 10. The chief conclusion to be drawn from this figure is that the measured performance of Heaters 9, 10 and 11 is consistent with the empirically-calculated performances of the comparable hypothetical elements. The temptation to draw further conclusions as to the effects of different grades of graphite must be resisted at this stage. These curves apply only to the case when α is constant and the same for all the elements, and because of the differences in ρ_o, this almost certainly implies different geometries. However, it is possible that the theoretical treatment given here might be extended in such a way as to suggest an optimum grade of graphite, if such an extension were desired.

5. CONCLUDING REMARKS

The experiments described in Part II of this report have yielded much information of the performance of graphite resistance heaters for use with nitrogen. Inevitably, because of the exploratory nature of most of the experiments, not all the results obtained are amenable to a detailed comparative analysis. Consequently, only those tests for which the basic conditions were most nearly identical have been analysed here. It has been shown that the results of these tests fit into a consistent theoretical pattern which offers promise of extended application.

NOMENCLATURE

a	coefficient of τ in assumed expression for β
b	coefficient of $\tau^{3/2}$ in assumed expression for β
c_{p_o}	value at room temperature T_o of specific heat of gas at constant pressure; 261.7 joules per pound mass ^oR. for nitrogen at 530^oR.
$f(\xi)$	function involved in description of temperature distribution along element
$g(\tau_E)$	function describing variation of resistivity of element material with local element temperature
h_t	specific total enthalpy of gas; joules per pound mass
h_o	specific enthalpy of gas at temperature T_o; joules per pound mass
i	current;amps
k	constant used in representation of $(h_t-h_o)/c_{p_o}T_o$ as a function of τ, 0.0185 for nitrogen when $T_o = 530^oR$.
ℓ	length of element; inches
m	mass flow of gas; pound mass per second
m_o	cold mass flow of gas; pound mass per second
p_t	total pressure of gas; pound weight per square inch
p_{t_o}	total pressure of gas in cold flow; pound weight per square inch
r	resistance of element; ohms
r_o	cold resistance of element; ohms
r_{min}	minimum resistance of element during one run; ohms
v	voltage across element; volts
x	distance along element measured from gas entrance; inches

A_E	cross-sectional area of element, excluding gas passage; square inches
A^*	area of orifice; square inches
A^*_o	area of orifice in cold flow; square inches
R	gas constant for unit mass of gas
T_t	total temperature of gas; ^oR.
T_o	room temperature, value of T_t in cold flow; ^oR.
T_E	maximum temperature along element; ^oR.
α	non-dimensional quantity defined such that $\nu = (1/\alpha - 1)\,\tau$
β	r/r_o
ζ_o	thermal conductivity of element material at room temperature; watts per inch ^oR.
η	efficiency of heating system
η_o	value of η in cold flow - initial efficiency of heating system
θ	dummy variable in $\varphi(\theta)$
$\varkappa$	$r_o i^2/m_o c_{p_o} T_o$
λ	$r i^2/m_o c_{p_o} T_o$
μ	$m(h_t - h_o)/m_o c_{p_o} T_o$
ν	rate of heat loss from element divided by $m_o c_{p_o} T_o$
ξ	x/ℓ
ρ	resistivity of element material; ohm inches
ρ_o	resistivity of element material at room temperature; ohm inches
σ	value of ξ at which $T_E = T_{E_{max}}$
τ	$(T_t - T_o)/T_o$
τ_E	$(T_E - T_o)/T_o$
$\tau_{E_{max}}$	$(T_{E_{max}} - T_o)/T_o$

$\varphi(\theta)$	function used in expressing special form of $f(\xi)$
χ	$\tau/\tau_{E_{max}}$
ψ	$(1 + \tau)^{\frac{1}{2}}\ r/r_{min}$
ω_0	ratio of pressure drop down element to total pressure of gas at room temperature
Γ	factor involved in expression for mass flow through a sonic orifice

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2. Woolley, H. W.: Thermodynamic Properties of Gaseous Nitrogen. NACA TN 3271, 1956.

3. Currie, L. M.; Hamister, V. C. and MacPherson, H. G.: The Production and Properties of Graphite for Reactors. Peaceful Uses of Atomic Energy, Vol. 8, United Nations, 1956.

4. Industrial Carbon and Graphite Products. Handbook, National Carbon Company, 1959.

A.1 List of Tests in Phase 1

Heater Number	Element Material	Element Geometry	Gas	Run Number	Whether Analysis Presented (see A.4)	Nominal p_t p.s.i.a.	m_o lbs.mass per sec.	ω_o	r_o ohms
1	NT0005	Regular	A	1	Yes	1000	0.0206	0.049	not eval.
2	"	"	"	1	No (9)	1000	0.0204	0.092	
			"	2	No (1)	not constant	0.0204	0.075	
3	"	"	"	1	No (1)	not constant	0.0201	0.066	
			"	2	No (3)	1000	0.0233	0.120	
4	"	"	"	1	No (6)	1000	0.0264	0.140	
			"	2	No (1)	not constant	0.0266	0.140	
			"	3	No (1)	not constant	0.0245	0.051	
5	"	"	"	1	No (1)	not constant	0.0273	0.049	
			"	2	No (1)	not constant	0.0269	0.044	
6	"	"	"	1	No (1)	not constant	0.0276	0.031	
7	"	"	"	1	No (10)	1000	0.0263	0.114	
			"	2	No (4)	1000	0.0211	0.074	
			"	3	No (4)	1000	0.0211	0.070	
			"	4	No (4)	1000	0.0180	0.056	
			"	5	No (4)	1000	0.0171	0.052	
8	"	"	"	1	No (1)	not constant	0.0274	0.059	
			"	2	No (1)	not constant	0.0277	0.083	
			"	3	No (1)	not constant	0.0193	0.023	
9	"	"	"	1	Yes	1000	0.0272	0.093	0.0507
			"	2	Yes	1000	0.0271	0.090	0.0507
			"	3	No (4)	1000	0.0268	0.078	
			"	4	No (4)	1000	0.0262	0.070	
			"	5	No (2)	1250	0.0305	0.061	
			"	6	No (2)	750	0.0196	0.036	
10	"	"	"	1	Yes	1000	0.0276	0.062	0.0550
			"	2	No (6)	1000	0.0271	0.055	
			"	3	No (2)	500	0.0134	0.054	
11	"	"	"	1	Yes	1000	0.0269	0.063	0.0533
			"	2	Yes	1000	0.0271	0.061	0.0533
			"	3	Yes	1000	0.0263	0.064	0.0533
			"	4	No (4)	1000	0.0244	0.053	

"Not constant" means that p_t was not controlled to be constant but was allowed to adjust itself as the current was changed.

Gas A was regular nitrogen.

A.2 List of tests in Phase 2

Heater Number	Element Material	Element Geometry	Gas	Run Number	Whether Analysis Presented (see A.4)	Nominal p_t p.s.i.a.	m_o lbs.mass per sec.	ω_o	r_o
12	NT0005	Regular	A	1	Yes	1000	0.0280	0.065	not evaluated
			"	2	No (3)	"	0.0249	0.115	
13	A.T.J.	Regular	"	1	Yes	"	0.0280	0.058	not evaluated
			"	2	No (4)	"	0.0253	0.045	
14	A.T.J.	Regular	"	1	Yes	"	0.0275	0.073	not evaluated
			"	2	No (5)	"	0.0252	0.075	
15	RT0003	Regular	"	1	Yes	"	0.0276	0.055	not evaluated
			"	2	Yes	"	0.0267	0.058	not evaluated
16	RT0004	Short	"	1	Yes	"	0.0268	0.065	not evaluated
			"	2	Yes	"	0.0267	0.067	not evaluated
			"	3	No (4)	"	0.0248	0.030	
17	RT0003	Short	"	1	Yes	"	0.0265	0.045	not evaluated
			"	2	No (5)	"	0.0249	0.045	
			"	3	No (4)	"	0.0228	0.039	
			"	4	No (4)	"	0.0228	0.039	
			"	5	No (4)	"	0.0228	0.038	
			"	6	No (4)	"	0.0226	0.038	
18	RT0003	Regular	"	1	No (8)	"	0.0242	0.071	
			"	2	No (5)	"	0.0224	0.066	
			"	3	No (3)	"	0.0172	0.085	
			"	4	No (5)	"	0.0172	0.074	
			"	5	No (5)	"	0.0164	0.064	
			"	6	No (8)	"	0.0246	0.112	
			"	7	Nc (8)	"	0.0246	0.110	
			"	8	No (4)	"	0.0246	0.097	
			"	9	No (4)	"	0.0245	0.066	
			"	10	No (4)	"	0.0252	0.053	
19	A.T.J.	Regular	"	1	No (6)	"	0.0250	0.076	
			"	2	No (7)	"	0.0248	0.074	
			"	3	No (7)	"	0.0246	0.075	
			"	4	No (7)	"	0.0251	0.075	
			"	5	No (11)	"	0.0249	0.076	

Between Runs 5 and 6 of Element 18, the element was removed from the tunnel, cleaned and rebaked, and the orifice was cleaned of deposit.

In Run 1 of Element 19, p_t was controlled to be constant at 1000, 750, 500 and 200 psia in succession; the quoted m_o refers to p_t 1000 psia.

A.3 List of tests in Phase 3

Heater Number	Element Material	Element Geometry	Gas	Run Number	Whether analysis presented (see A.4)	Nominal p_t p.s.i.a.	m_o lbs. mass per second	
20	RT0003	Regular	B	1	No (6)	1000	0.0222	0.099
				2	No (12)		0.0227	0.096
				3	No (5)		0.0234	0.100
				4	No (5)		0.0240	0.090
21	A.T.J.	Regular	B	1	No (7)		0.0239	0.057
				2	No (7)		0.0241	0.058
22	A.T.J.	Thick	B	1	No (7)		0.0246	0.062
				2	No (7)		0.0246	0.062
				3	No (4)		0.0248	0.034
23	A.T.J.	Short	C	1	No (7)		0.0246	0.057
24	A.T.J.	Short	C	1	No (12)		0.0267	0.070

Gas B was regular nitrogen with decreased water content

Gas C was super-high purity nitrogen

A.4 Reasons governing selection of tests for analysis

The reasons for the omission from the analysis of the results of certain tests are as follows:

(1) Gas total pressure not constant during run

(2) Gas total pressure constant but not equal to 1000 psia

(3) Pressure drop down cold element greater, by more than 10 percent, than value in previous run of same element

(4) Pressure drop down cold element less, by more than 10 percent, than value in first run of same element

(5) Cold mass flow different, by more than 5 percent, from value in first run of same element

(6) Fewer than three current settings

(7) Highest gas total temperature attained not significantly greater than $1500^{\circ}R$.

(8) Cold mass flow different, by more than 5 percent, from value in otherwise comparable tests of other elements

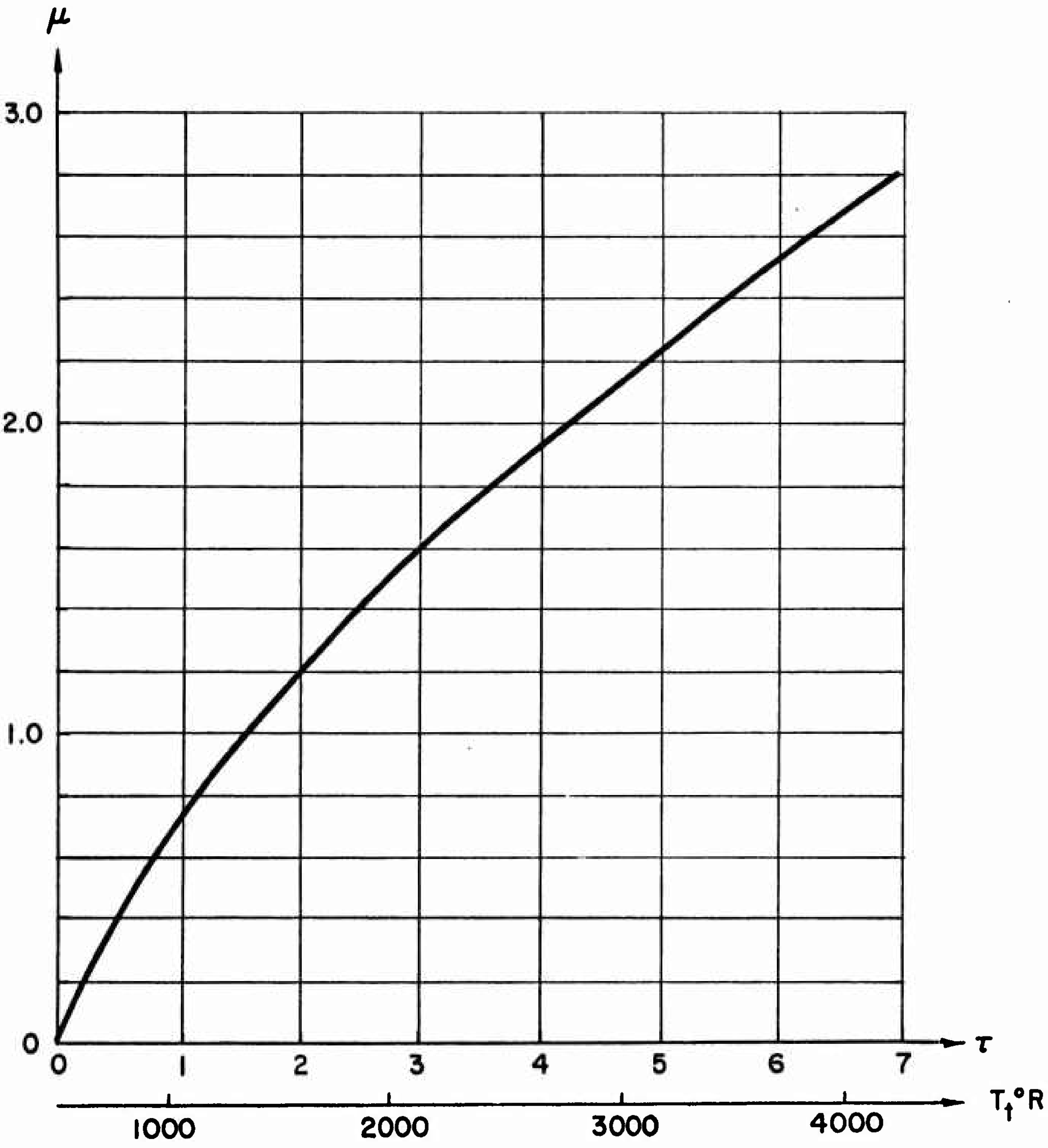
(9) Severe oil contamination during run

(10) Pressure drop down cold element particularly high

(11) Element failed early in run

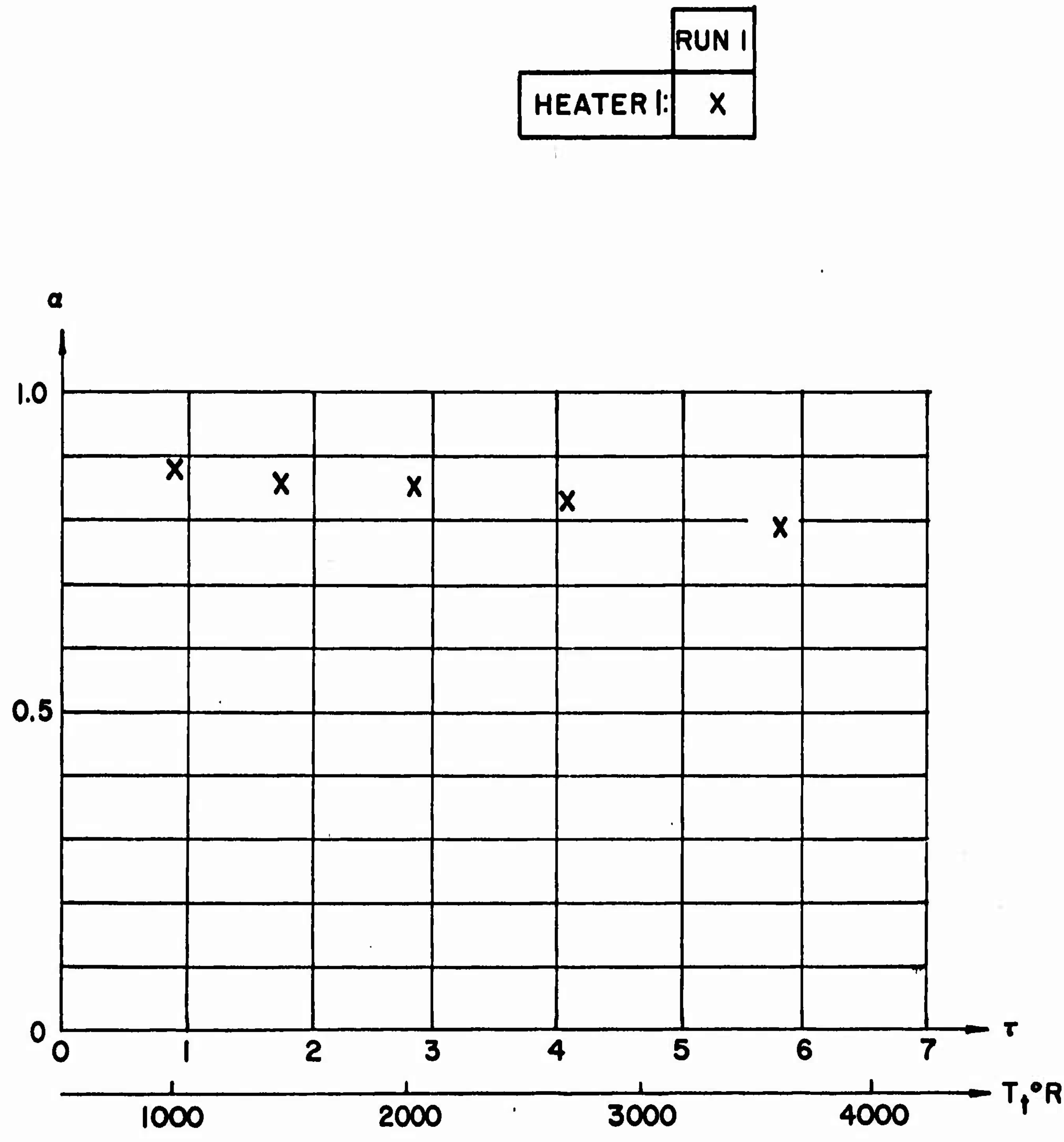
(12) Isolated cases without comparative results

Note: Only one reason is given for the elimination from presentation of any one run; the reason given is simply the first apparent, and many runs could be eliminated for other reasons also.



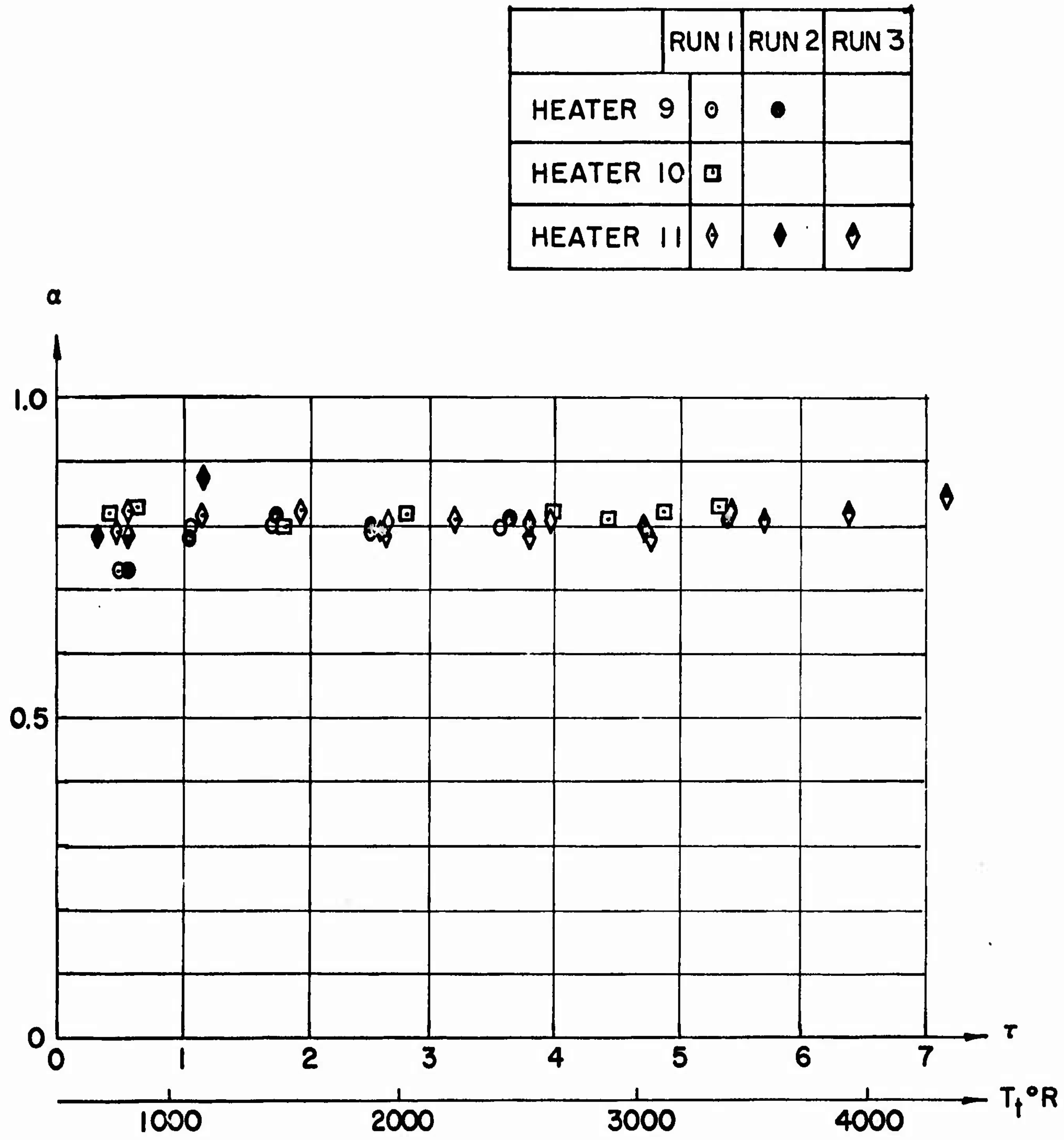
XIII B-1

FIG. 1 TOTAL ENTHALPY FUNCTION μ FOR NITROGEN



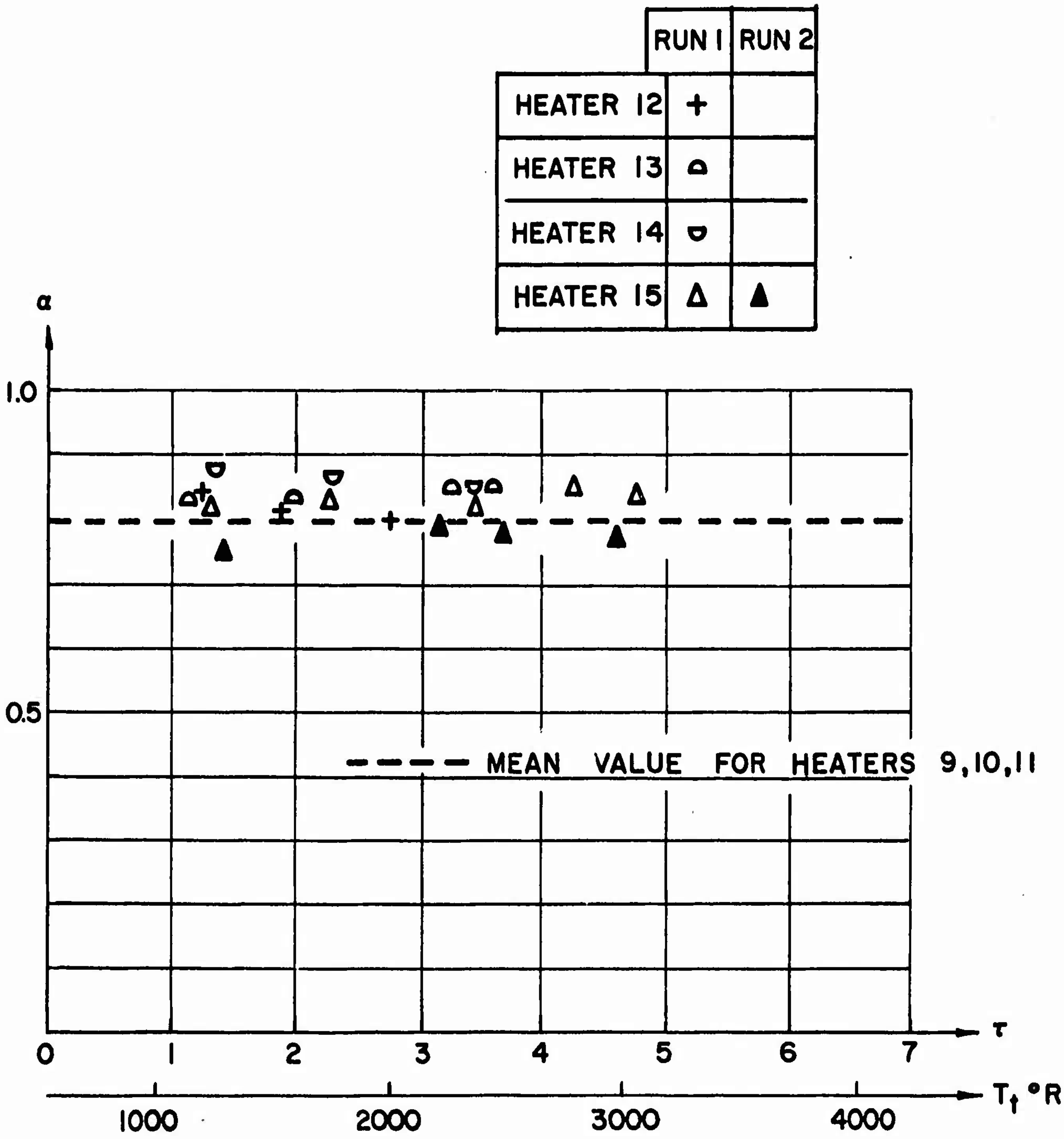
XIII B-2

FIG. 2 VARIATION OF α WITH τ FOR HEATER 1



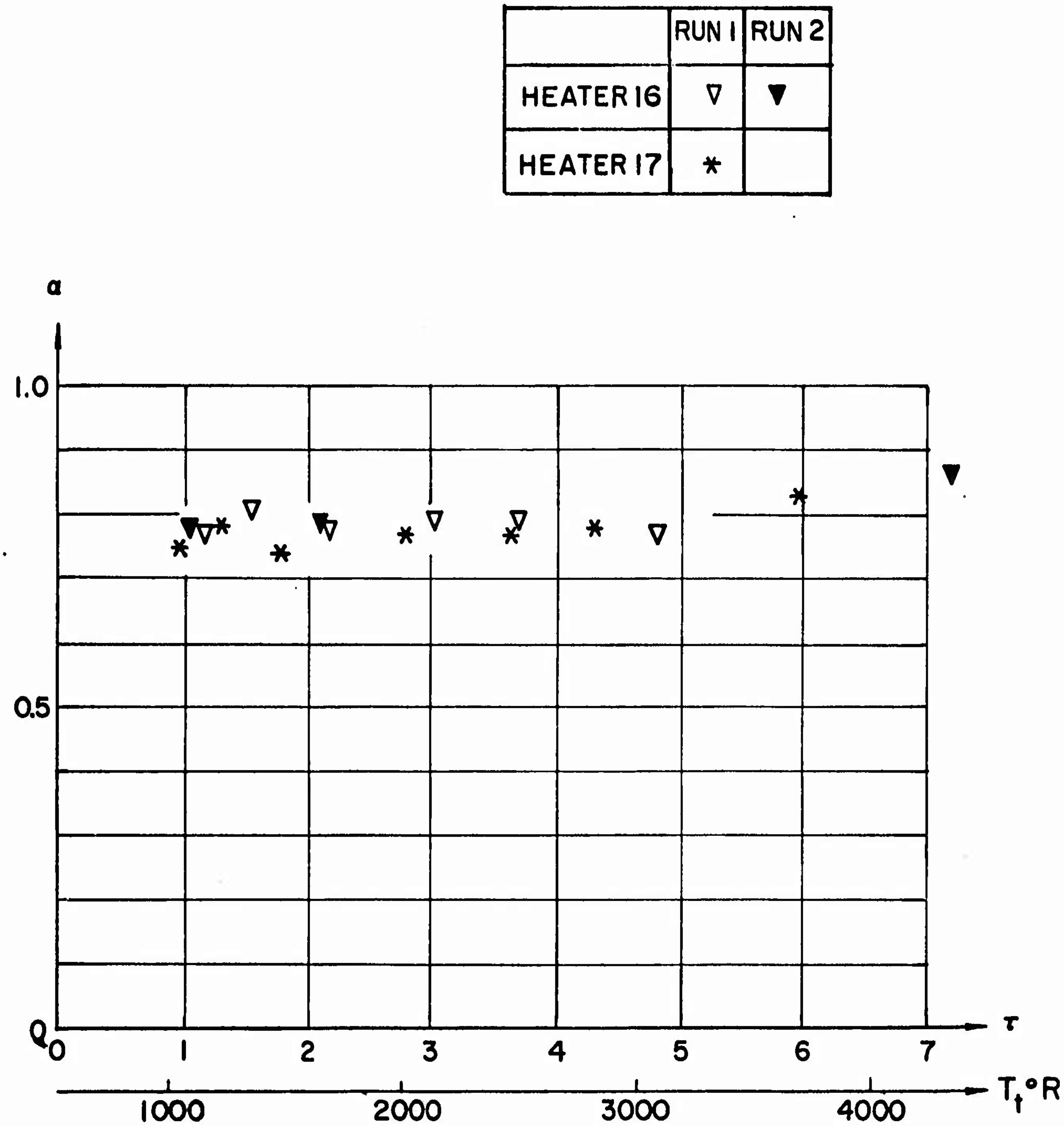
XIII B-3

FIG. 3 VARIATION OF α WITH τ FOR HEATERS 9,10,11



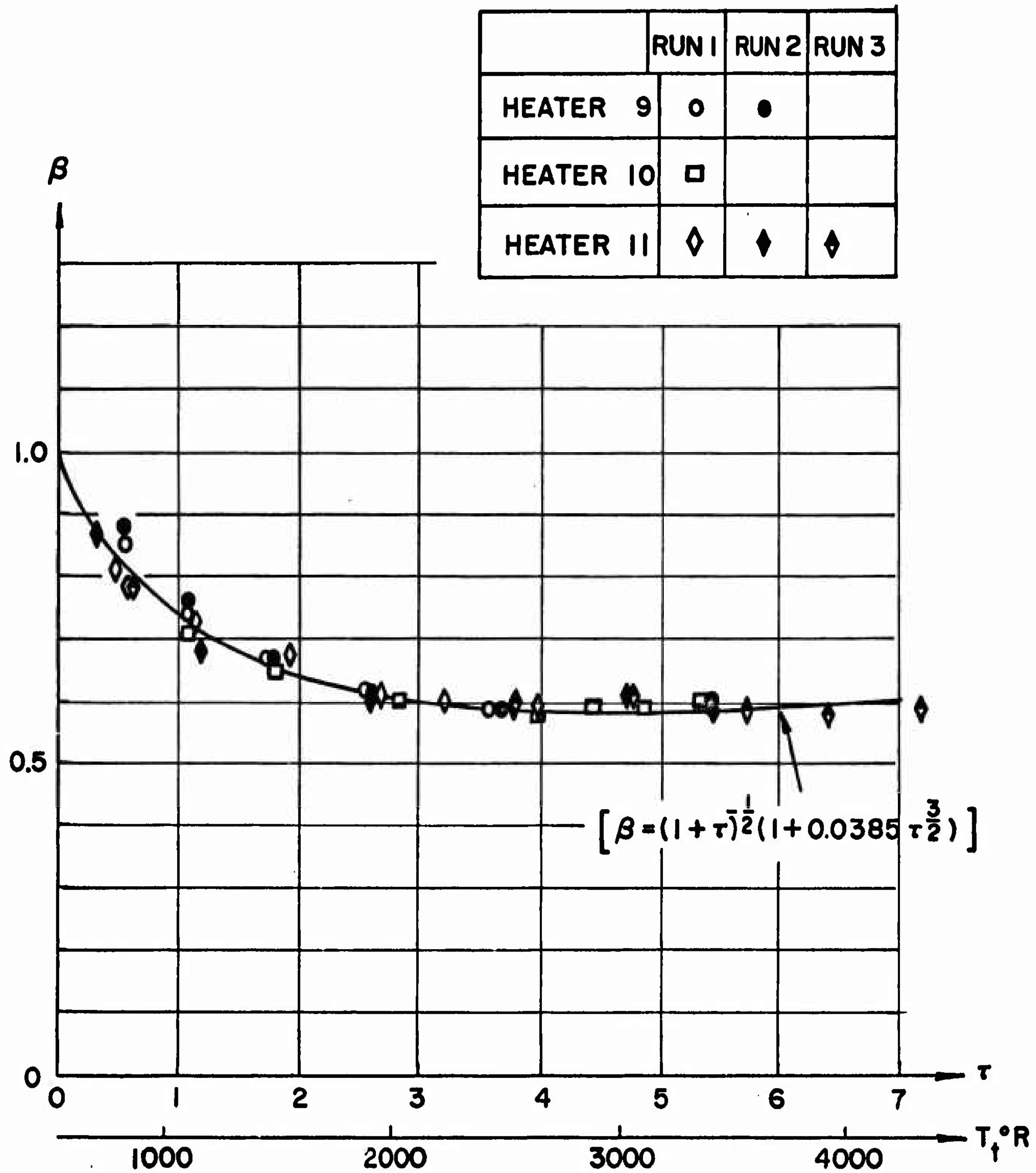
XIII B-4

FIG. 4 VARIATION OF α WITH τ FOR HEATERS 12,13,14,15



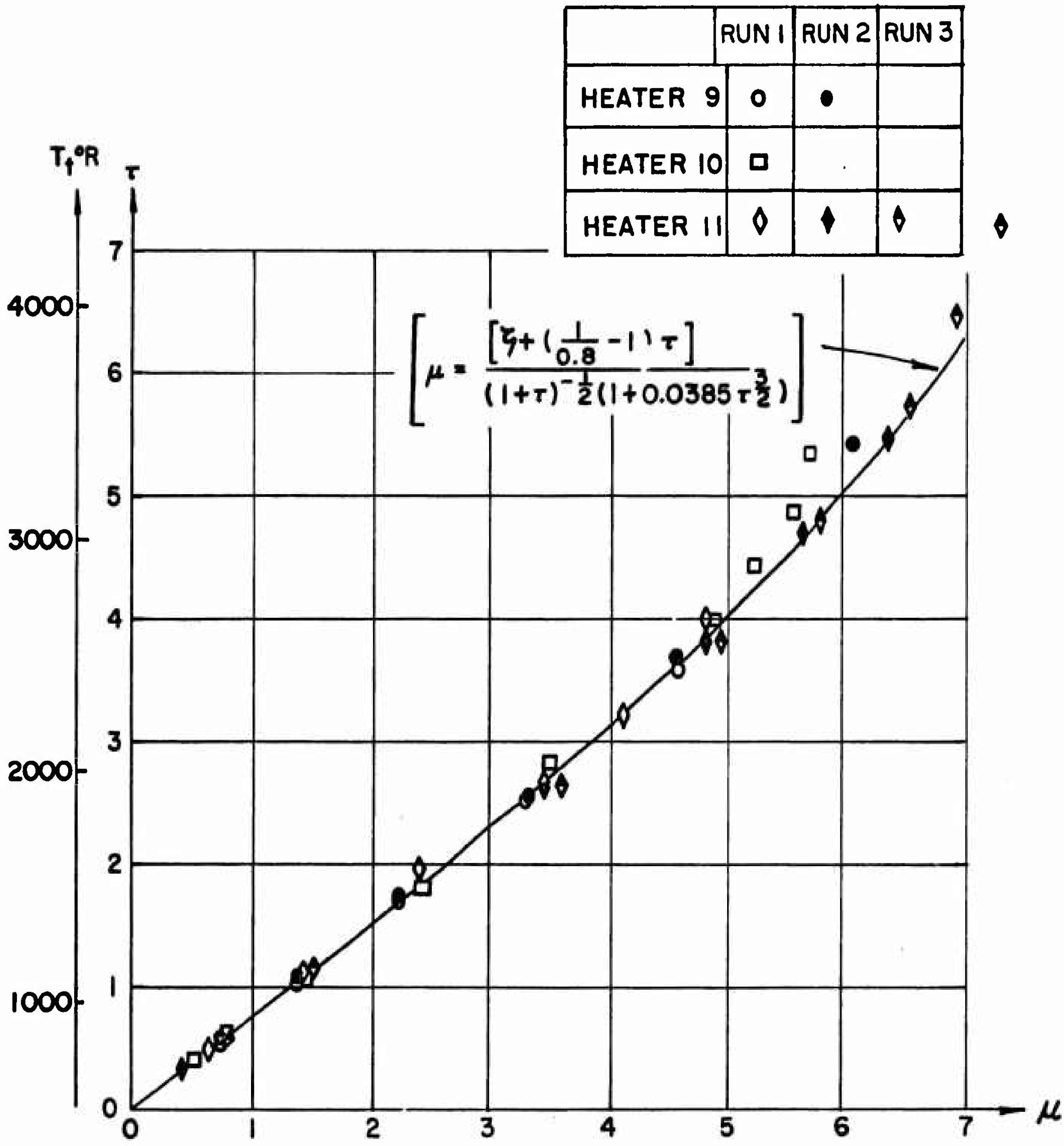
XIII B-5

FIG. 5 VARIATION OF a WITH τ FOR HEATERS 16,17



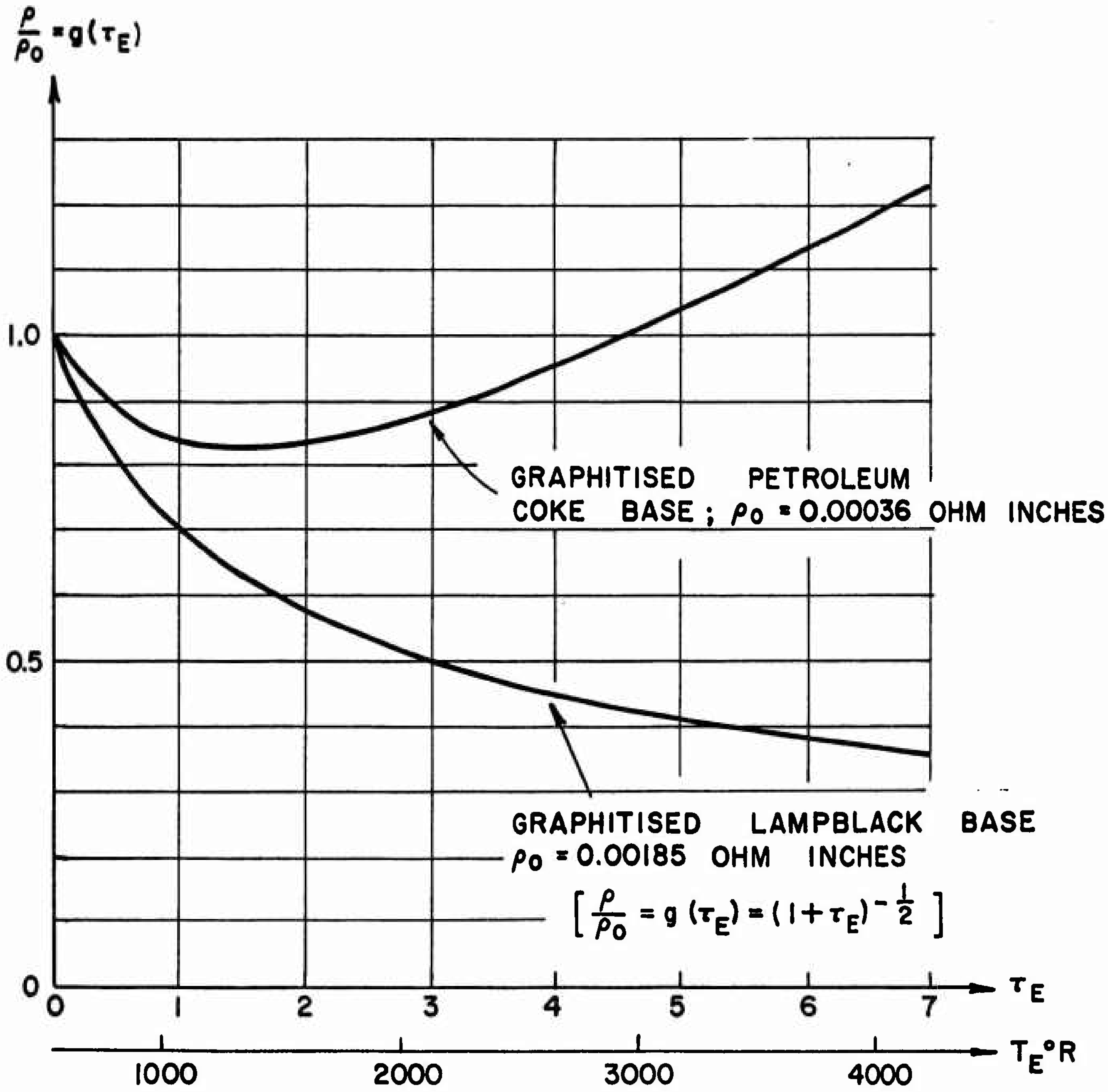
XIII B-6

FIG. 6 VARIATION OF β WITH τ FOR HEATERS 9,10,11



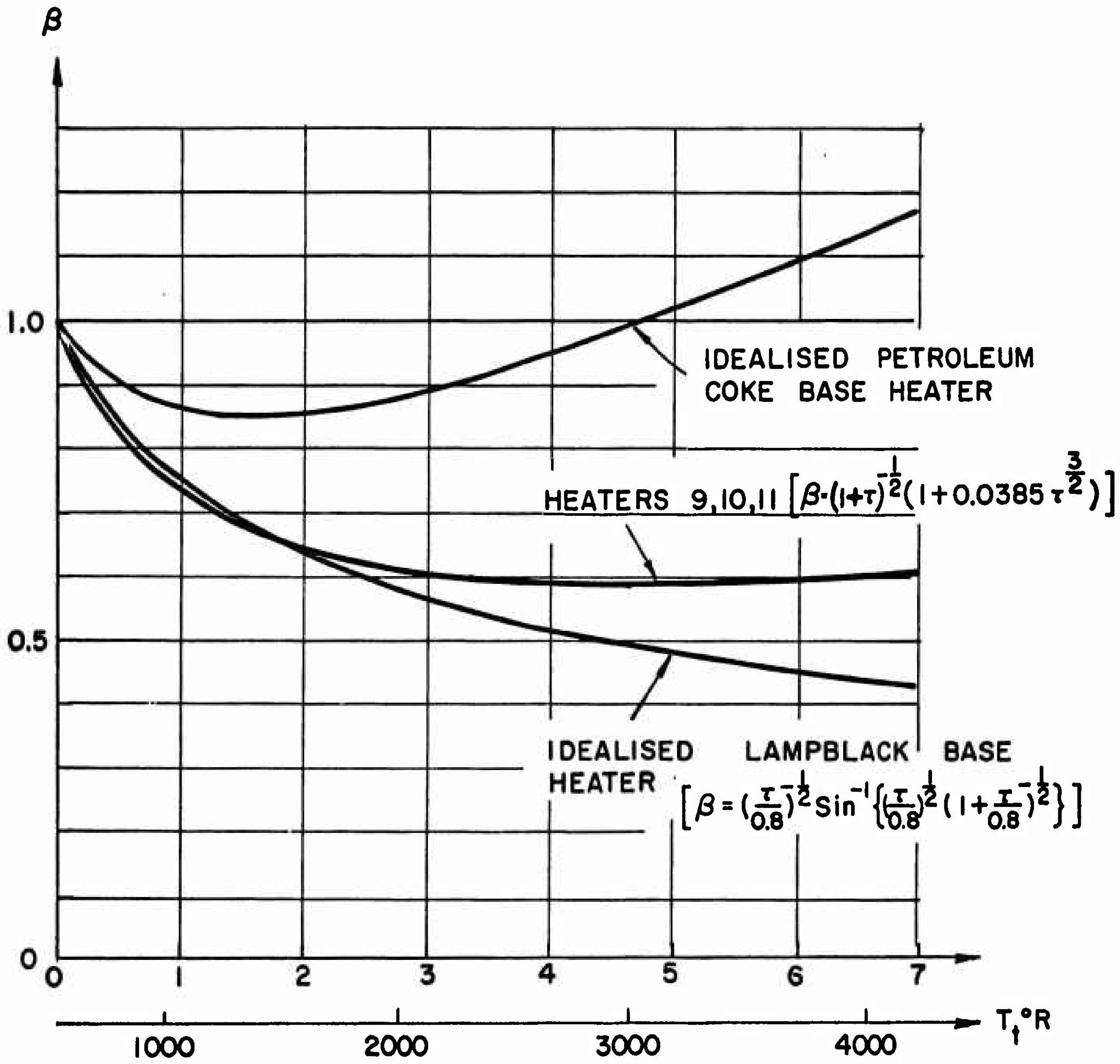
XIII B-7

FIG.7 VARIATION OF τ WITH μ FOR HEATERS 9,10,11



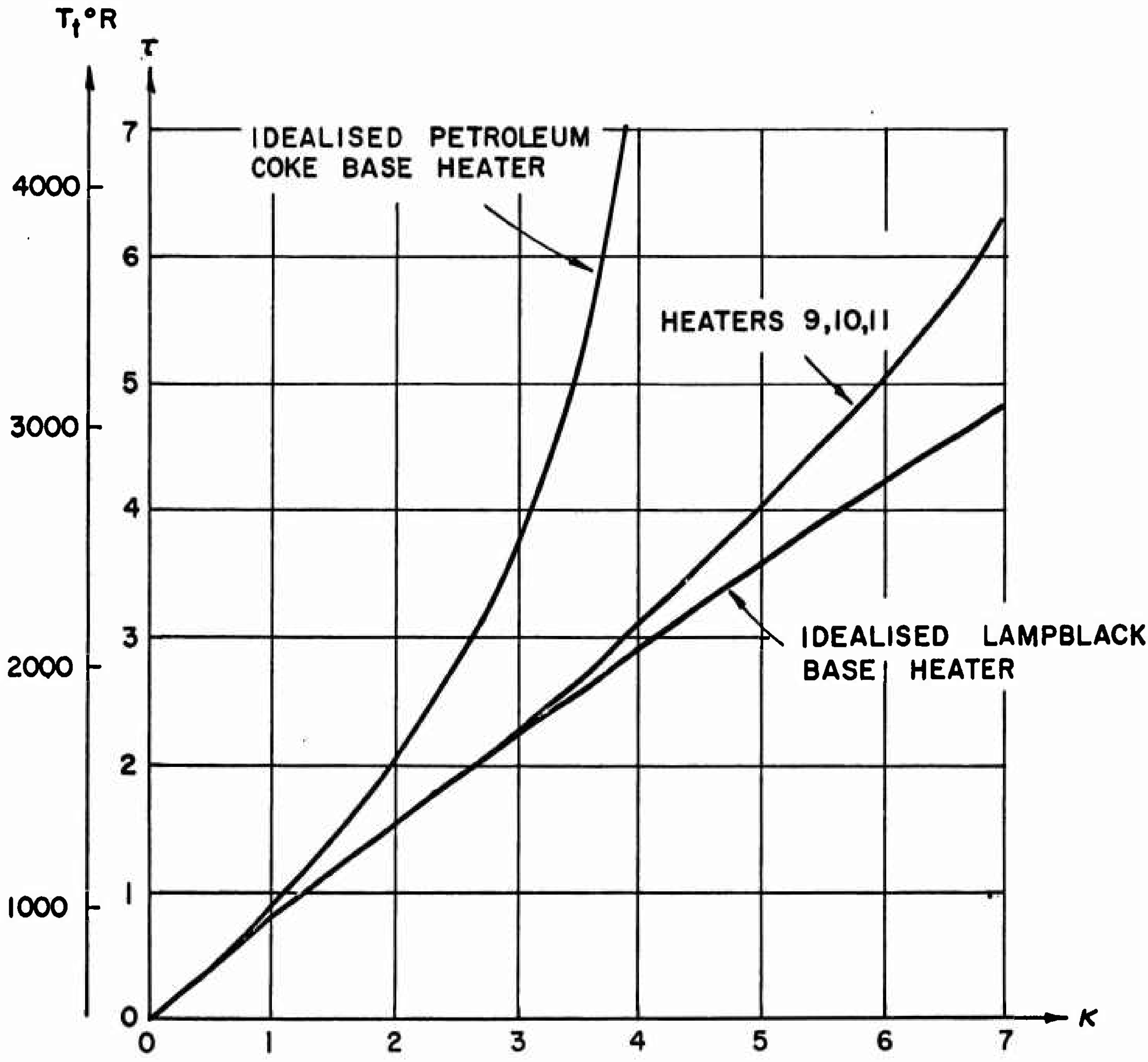
XIII B-8

FIG. 8 VARIATION OF RESISTIVITY FOR TWO GRADES OF GRAPHITE.



XIII B-9

FIG. 9 COMPARISON OF VARIATION OF β WITH τ FOR HEATERS 9,10,11 AND TWO HYPOTHETICAL IDEALISED HEATERS.



XIII B-10

FIG. 10 COMPARISON OF VARIATION OF τ WITH κ FOR HEATERS 9, 10, 11 AND TWO HYPOTHETICAL IDEALISED HEATERS.